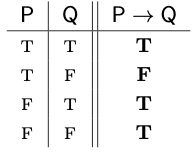
**Ch 7 - Conditionals**

* a connective is truth functional if the truth or falsity of compound statements made with it is completely determined by the truth values of its constituents
  + in other words, its meaning can be captured by a truth table
* FOL does not include connectives that are not truth functional
* **because** is not truth functional
  + typically asserts some sort of causal connection between the facts described by the constituent sentences
* we will now introduce **two new truth-functional connectives**
  + **material conditional**
  + **material biconditional**
* these are standard features of FOL, but do not actually increase the expressive power of FOL; they do make it much easier to say and prove certain things, however

**material conditional symbol**

* **material conditional** P → Q
* **antecedent** P
* **consequent** Q



* another way of saying ~P | Q
* a material conditional with a false antecedent is always true
* if the statement P → Q is true, then you know that either P and Q are true, or P is false, in which case Q can be either true or false; the short way to say this is ~P | Q, ie either P is false, or Q is true

**if…then**

* “If P then Q” comes fairly close to translating P → Q, and is how we will translate material conditional statements
* however, in English there are sentences that use if…then but don’t translate correctly to material conditional connective
  + *If Max had been home, then Carl would have been there too.*
  + It is possible for this sentence to be false even if “Max had been at home” is false. If it is not the case that Carl would have been home too, because Carl was actually somewhere else the whole time, “Carl would have been there too” is false.
* if…then can also be used to express a sort of causal connection between antecedent and consequent
  + as a result, many uses of if…then in English aren’t truth functional
    - the truth of the whole depends on something more than the truth values of the parts; it depends on there being some genuine connection between the subject matter of the antecedent and the consequent
  + *If the book is here then the mailman delivered it.*
    - What if the book is a present from someone who brought it and left it here? Then the consequent is false. But if the mailman did deliver it then the consequent is true.
    - What if the book is not here, and the mailman delivered it, but the book was stolen from the mailbox? The sentence as a whole is false, because it is not the case that just because the book is here it was delivered by the mailman.
* **other English expressions translated using material conditional**
  + P only if Q: P → Q
  + Q provided P
  + Q if P
* **P only if Q** is translated to FOL as P → Q, but **P if Q** is translated as Q → P
* **only if** introduces a **necessary condition**, a condition that must hold in order for something else to obtain
  + **“you will pass the course only if you turn in all the homework assignments”**
  + homework is a necessary condition for passing; if you don’t do it you won’t pass, but there is no guarantee you will pass if you do turn in homework.
  + this statement is false only when “you pass the course” is true and “you turn in all the homework” is false
* **if** introduces a **sufficient condition**, one guaranteeing that something else will obtain
  + **“you will pass the course if you turn in all the homework”**

**unless P, Q** or **Q unless P**

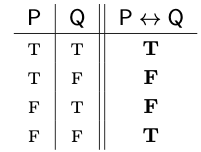
* **true in the same circumstances as Q if not P, translated as ~P→Q**
* “Claire is at the library unless Max is home”
* “Claire is at the library if Max is not home”
* replace “unless” with “if not”

**reduce logical consequence to logical truth when we have finitely many premises**

* Q is a logical consequence of P1, …, Pn if it is impossible for the premises to all be true and Q false
* if we consider “premises to all be true” as the conjunction of all the premises being true, then it is impossible for the conjunction to be true and Q false.
* this means that Q if P1 & … & … Pn
* ie, if Q is a logical consequence of P1, …, Pn then the sentence P1 ^ … ^ … Pn → Q is a logical truth

**Biconditional Symbol** ↔

* is true if and only if P and Q have the same truth value
* aka “just in case”
* Even(n) ⟺ Even(n^2)
* important fact
  + P and Q are logically equivalent if and only if P ↔ Q is a logical truth
  + note that P ⟺ Q means that P and Q are logically equivalent
  + Therefore, what we’ve said here is that
    - P ⟺ Q is true iff FOL sentence P ↔ Q is logically necessary



**truth-functional completeness**

* we have so far five truth-functional connectives
  + one unary (not) and four binary (and, or, material conditional, material biconditional)
* are there English expressions that are truth functional but cannot be expressed using the symbols we have so far?
  + consider first binary connectives
  + there are 2^4 possible truth tables for binary connectives, hence there are 16 possible distinct truth-functional binary connectives that we could define
* it is possible to show that we can express each of the 16 truth-functional connectives with just the symbols we have so far
* furthermore, we can write each sentence in disjunctive normal form
* we can do the same thing for ternary connectives
  + there are 2^8 = 256 possible truth tables, and we can express each one as a sentence in disjunctive normal form
* in general, any truth function, of any arity whatsoever, can be expressed using just the Boolean connectives not, and, and or.
* **a set of connectives is truth-functionally complete if the connectives in the set allow us to express any truth function**
* **Theorem:** the boolean connectives &, |, and ~ are truth-functionally complete
* there are other sets of operators that are truth-functionally complete
  + ~, and
  + ~, or
  + neither or